Differential Equation Summary

1. **Solve DE** (a) $\frac{dy}{dx} = f(x)$

$\Rightarrow y = \int f(x)\,dx$

(b) $\frac{dy}{dx} = f(y)$

$\Rightarrow \int \frac{1}{f(y)}\,dy = \int 1\,dx$

(c) $\frac{d^2y}{dx^2} = f(x)$

$\Rightarrow \frac{dy}{dx} = \int f(x)\,dx \Rightarrow$ integrate again wrt x to get y.

(d) Use a substitution to reduce the DE to a simpler form

*Not in the syllabus but it is good to know too:

Solve $\frac{dy}{dx} = f(y)g(x)$

$\Rightarrow \int \frac{1}{f(y)}\,dy = \int g(x)\,dx$

2. For first order DE like (a) and (b), there is only one arbitrary constant in the general solution whereas there are two constants for second order DE like (c).
Initial conditions are given to enable the constants to be found.

3. If DE is solved by substitution method, chain rule is used.
It is important to remember to change your final answer back to the original variables.

4. The general solutions when sketched form a family of curves.
If you are asked to sketch a family of curves, we usually sketch 3 members to represent 3 members of different types.
Curves should contain initial conditions, asymptotes, stationary points or axial intercepts if any. This depends very much on the requirement of the question.

5. Modeling of problem using DE.

Note that the rate of change of y is denoted by $\frac{dy}{dt}$.

A common model:

V is the volume of water in a tank.
Water is flowing into the tank at a rate that is proportional to the volume of water in the tank. Water is flowing out of the tank at a constant rate of 3 m^3s^{-1}. Form a differential equation.

Ans: **Usually we use** $\frac{dV}{dt} = \frac{dV_{in}}{dt} - \frac{dV_{out}}{dt} = kV - 3$

6. Note that if the rate is decreasing at 2 ms^{-1}, $\frac{dV}{dt} = -2$. The rate expression will carry a negative sign.