

CHECKLIST FOR INTEGRATION TECHNIQUES

Standard Forms:

$$\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

By Parts:

$$\int u v' dx = u v - \int u' v dx, \quad \text{where } v' = \frac{dv}{dx} \text{ and } u' = \frac{du}{dx}.$$

Can you integrate these?

1. (i) $\int \cos^2 x dx$ (ii) $\int \sin^2 x dx$
2. (i) $\int \sec^2 x dx$ (ii) $\int \tan^2 x dx$
3. (i) $\int \sec x dx$ (ii) $\int \tan x dx$ (iii) $\int \sec x \tan x dx$
4. (i) $\int \sin^2 x \cos x dx$ (ii) $\int \tan^3 x \sec^2 x dx$
5. $\int \sin 4x \cos 3x dx$
6. (i) $\int \frac{x^2}{x^2-1} dx$ (ii) $\int \frac{1}{(x-1)(x^2+1)} dx$
7. (i) $\int \frac{1}{\sqrt{4-9x^2}} dx$ (ii) $\int \frac{1}{\sqrt{8-2x-x^2}} dx$
8. (i) $\int \frac{1}{4+9x^2} dx$ (ii) $\int \frac{1}{8-2x-x^2} dx$
9. (i) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ (ii) $\int \frac{x}{\sqrt{1-x^2}} dx$
10. (i) $\int \frac{x+3}{\sqrt{4-x^2}} dx$ (ii) $\int \frac{x}{x^2-2x+3} dx$
11. (i) $\int (\ln x)^2 dx$ (ii) $\int e^x \sin x dx$ (iii) $\int \sin^{-1} 2x dx$
12. $\int x^3 e^{x^2} dx$
13. $\int a^x dx$

Detailed solutions at <http://ayec.com.sg/useful-resources/>

Answers:

1. (i) $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$ (ii) $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$
2. (i) $\tan x + C$ (ii) $\tan x - x + C$
3. (i) $\ln|\sec x + \tan x| + C$ (MF26) (ii) $\ln|\sec x| + C$ (MF26) (iii) $\sec x + C$
4. (i) $\frac{1}{3}\sin^3 x + C$ (ii) $\frac{1}{4}\tan^4 x + C$
5. $-\frac{1}{14}(\cos 7\theta + 7\cos \theta) + C$ (Use Factor Product formula from MF26)
6. (i) $x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$ (ii) $\frac{1}{2}\ln|x-1| - \frac{1}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x + C$
7. (i) $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$ (ii) $\sin^{-1}\left(\frac{x+1}{3}\right) + C$
8. (i) $\frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) + C$ (ii) $\frac{1}{6}\ln\left|\frac{4+x}{2-x}\right| + C$
- 9 (i) $2e^{\sqrt{x}} + c$ (ii) $-\sqrt{1-x^2} + C$
- 10 (i) $-\sqrt{4-x^2} + 3\sin^{-1}\left(\frac{x}{2}\right) + C$ (ii) $\frac{1}{2}\ln|x^2-2x+3| + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C$
- 11 (i) $x(\ln x)^2 - 2x\ln x + 2x + C$ (ii) $\frac{1}{2}e^x(\sin x - \cos x) + C$
(iii) $x\sin^{-1}2x + \frac{1}{2}\sqrt{1-4x^2} + c$
12. $\frac{1}{2}xe^{x^2} - \frac{1}{2}e^{x^2} + C$
13. $\frac{a^x}{\ln a}$