Polynomial & Long Division

Basic Notes:

To divide $P(x) = 3x^3 + x - 4$ by $S(x) = x + 2,$ we arrange the dividend $P(x)$ and the divisor $S(x)$ in the following way:

$$x + 2\overline{\smash{)}\,3x^3 \qquad + x - 4}$$

Note that an empty space is left for the missing x^2 in $P(x)$.

Dividing the first term of the dividend (i.e., $3x^3$) by the first term of the divisor (i.e., x) yields the term $3x^2$. We then multiply $3x^2$ by $x + 2$ and subtract the product from the dividend as follows:

$$\begin{array}{rl}
 & 3x^2 \\
x + 2 \big) & \overline{3x^3 \qquad\qquad + x - 4} \\
 & \underline{3x^3 + 6x^2 \qquad\qquad} \\
 & \qquad -6x^2 + x - 4
\end{array}$$

Now we divide the resulting $-6x^2 + x - 4$ by $x + 2$ and proceed as before to get the following:

$$\begin{array}{rl}
 & 3x^2 \qquad -6x + 13 \\
x + 2 \big) & \overline{3x^3 \qquad\qquad + x - 4} \\
 & \underline{3x^3 + 6x^2 \qquad\qquad} \\
 & \qquad -6x^2 + x \quad -4 \\
 & \qquad \underline{-6x^2 - 12x \qquad} \\
 & \qquad\qquad 13x \quad -4 \\
 & \qquad\qquad \underline{13x \ + 26} \\
 & \qquad\qquad\qquad \underline{-30}
\end{array}$$

The procedure ends at this point since the remainder (-30) is 1 degree lower than the divisor $x + 2$.

The above result may be expressed as

$$\frac{3x^3 - 6x + 13}{x + 2} = (3x^2 - 6x + 13) - \frac{30}{x + 2}.$$

Worksheet: Polynomial & Long Division

1(i) Find the coefficient of the x term and the constant term in the product $(2x^3 + x^2 - 4x + 1)(x^2 - 5x + 3)$.

(ii)Find the coefficient of the x^3 term in the product $(2x^2 - x + 7)(x^2 + 7x - 3)$.

(iii) Find the coefficient of the x^4 term in the product $(-3x^4 + 7x^2 + 1)(3x^2 - 7)$.

2. In each of the following, find the quotient and remainder when $P(x)$ is divided by $D(x)$:

(i) $P(x) = 4x^2 + 3x - 7$, $D(x) = x - 2$;

(ii) $P(x) = x^5 + x^4 + x^3 + x^2 + 1$, $D(x) = x^2 - 1$;

(iii) $P(x) = x^4 + 9x^3 + 1$, $D(x) = x^2 + x + 1$

3. Solve the equation $x^4 + 5x^2 - 2 = 0$, giving your answers to 3 significant figures.

Answers:

(1) (i) $-17; 3$; (ii) 13; (iii) 42

(2)(i) $4x + 11, 15$; (ii) $x^3 + x^2 + 2x + 2$, $2x + 3$; (iii) $x^2 + 8x - 9$, $x + 10$

(3) ± 0.610