APGP Word Problems

For self-practice. Solutions can be found at www.ayec.com.sg under useful resources.

1(a) On 1 January 2014, Mr. Spendalot uses a credit card to borrow $2000 from a bank, at an interest rate of 2% a month. He repays the bank $50 on the 10^{th} of each month, starting from January. Interest is always charged on the balance at the end of each month.

(i) Find the outstanding amount at the end of n^{th} month. [3]

(ii) How many months does he take to repay the entire loan? [2]

(b) Mr Thrift makes use of a special offer from a bank to obtain an interest-free loan of $2000. He decides to pay $50 in the first month. On the first day of each subsequent month, he pays $10 more than in the previous month. How many complete months would it take for him to fully repay the debt? [3]

[Answers: (a) (i) $2550-550\left(1.02^n\right)$ (ii) 78 (b) 16]

2. A bank offers a cash loan of $10,000. To make the loan attractive, the bank offers the following repayment plan.

Repay a fixed amount of $$x$ to the bank on the 15^{th} of every month. At the end of each month, the bank will add an interest at a fixed rate of 5% on the remaining amount owed. When the amount owed is less than $$x$, only the balance will have to be paid on the 15^{th} of the following month.

John takes up the loan on 1^{st} October 2012.

(i) How much will he owe the bank on 31^{st} October 2012 after the interest has been added? Leave your answer in terms of x. [1]

(ii) Show that the total amount of money John owes the bank at the end of n months is given by $\$\left[10000\left(1.05^n\right)-21x\left(1.05^n-1\right)\right]$. [3]

(iii) If John repays $500 every month to the bank, find the total number of months for the loan to be repaid fully. [3]

[Answers: (i) $\$\left[\left(10000-x\right)\left(1.05\right)\right]$ (iii) 63]

3. A fund is established with a single deposit of $2500 at the beginning of 2011 to provide an annual bursary of $150. The fund earns interest at 3.5% per annum, paid at the end of each year.

If the first bursary is awarded at the end of 2011 after interest is earned, show that at the end of n years, the amount (in dollars) remaining in the fund is

$$\frac{-12500}{7}\left(1.035\right)^n+\frac{30000}{7},$$

When is the last year that the bursary can be awarded? **[6]**

[Answer: 2035]

Solutions to Q1

(a)

(i) Balance at end of 1 month $= 1.02(2000 - 50)$

Balance at end of 2 months $= 1.02[1.02(2000 - 50) - 50] = 1.02^2(2000) - 50(1.02^2 + 1.02)$

Balance at end of 2 months

$= 1.02[1.02^2(2000) - 50(1.02^2 + 1.02) - 50] = 1.02^3(2000) - 50(1.02^3 + 1.02^2 + 1.02)$

Balance at end of *n* months

$= 1.02^n(2000) - 50(1.02^n + 1.02^{n-1} \ldots + 1.02)$

$= 1.02^n(2000) - 50\left(\dfrac{1.02(1.02^n - 1)}{1.02 - 1}\right)$

$= 1.02^n(2000) - 2500\big(1.02(1.02^n - 1)\big)$

$= 1.02^n(2000) - 2550\big((1.02^n - 1)\big)$

$= 1.02^n(2000) - 1.02^n(2550) + 2550$

$= 2550 - 1.02^n(550)$

(ii) loan repaid ⇒ balance ≤ 0 ⇒ $2550 - 1.02^n(550) \leq 0 \Rightarrow n \geq 77.46$ so he takes 78 months.

(b)

Let *n* be the no. of months. Then

$\dfrac{n}{2}(2a + (n-1)d) \geq 2000$

$\Rightarrow \dfrac{n}{2}(100 + (n-1)10) \geq 2000$

$\Rightarrow \dfrac{n}{2}(90 + 10n) \geq 2000$

$\Rightarrow n^2 + 9n - 400 \geq 0$

Solving, $n \leq -25, n \geq 16$.

16 complete months to fully repay the debt.

Solutions to Q2

(i)

On the 31st Oct 2012 (at the end of 1st month), the amount John owes the bank

$= \$\left[(10000 - x)(1.05)\right]$ or $= \$\left[10000(1.05) - x(1.05)\right]$

(ii)

At the end of 2nd month, the amount John owes the bank

$= \$\left[10000(1.05) - x(1.05) - x\right](1.05)$

$= \$\left[10000(1.05)^2 - x(1.05)^2 - x(1.05)\right]$

At the end of 3rd month, the amount John owes the bank

$= \$\left[10000(1.05)^2 - x(1.05)^2 - x(1.05) - x\right](1.05)$

$= \$\left[10000(1.05)^3 - x(1.05)^3 - x(1.05)^2 - x(1.05)\right]$

At the end of *n*th month, the amount John owes the bank

$= \$\left[10000(1.05)^n - x(1.05)^n - \cdots - x(1.05)^2 - x(1.05)\right]$

$= \$\left[10000(1.05)^n - x[(1.05)^n + \ldots + (1.05)^2 + (1.05)]\right]$

$= \$\left[10000(1.05)^n - x\dfrac{1.05\left(1.05^n - 1\right)}{1.05 - 1}\right]$

$= \$\left[10000(1.05)^n - 21x\left(1.05^n - 1\right)\right]$

(ii) Let $x = 500$,

amount owed by end of the nth months = $10000(1.05)^n - 21(500)\left(1.05^n - 1\right)$

For the loan to be repaid fully, amount owed should be 0.

$10000(1.05)^n - 21(500)\left(1.05^n - 1\right) = 0$

$10000(1.05)^n - 10500\left(1.05^n\right) + 10500 = 0$

$10500 = 500\left(1.05^n\right)$

$1.05^n = 21$

$n = \dfrac{\ln 21}{\ln 1.05} = 62.4$

At n = 62, the amount owed is not 0 yet. So need 63 months.

$\therefore$ the number of complete months required is 63.

Solutions to Q3

Amount of money in fund after giving out *n* years of bursary:

$n=1$: $(1.035)(2500) - 150$

$n=2$: $(1.035)\,[(1.035)2500 - 150] - 150$
$= (1.035)^2(2500) - (1.035)150 - 150$

at *n*th year: $(1.035)^n(2500) - (1.035)^{n-1}(150) - (1.035)^{n-2}(150) - \ldots - 150$

$$= (1.035)^n(2500) - (150)[1 + (1.035) + (1.035)^2 + \ldots + (1.035)^{n-1}]$$

There are n terms here!

$$= (1.035)^n(2500) - 150\left(\frac{1.035^n - 1}{0.035}\right)$$

$$= (1.035)^n(2500) - \frac{150}{0.035}\left((1.035)^n - 1\right)$$

$$= (1.035)^n(2500) - \frac{30000}{7}\left((1.035)^n - 1\right)$$

$$= (1.035)^n\left[2500 - \frac{30000}{7}\right] + \frac{30000}{7}$$

$$= -\frac{12500}{7}(1.035)^n + \frac{30000}{7}$$

Amount of money in fund after giving out *n* years of bursary ≥ 0

$$\frac{30000}{7} - \frac{12500}{7}(1.035)^n \geq 0$$

$$(1.035)^n \leq \frac{30000}{12500}$$

$$n \leq 25.44$$

$$n = 25\,years$$

2011 – first year and 2035 – 25th year
Last year is 2035.