Summary for APGP

	Arithmetic Progression (AP)	Geometric Progression (GP)
Formula for term u_n	$u_n = a + (n-1)d$	$u_n = ar^{n-1}$
Formula for sum S_n	$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ $= \frac{n}{2}\left(a + l\right)$	$S_n = \frac{a\left(1-r^n\right)}{1-r}$ or $\frac{a\left(r^n-1\right)}{r-1}$
Is sum (series) convergent?	$n \to \infty, S_n \to \infty$ or $-\infty$ [Except for special case when $d = 0$.] Arithmetic series is not convergent.	$\lvert r \rvert < 1 \Leftrightarrow S_\infty = \frac{a}{1-r}$ exist $\Leftrightarrow$ Geometric series is convergent
Show $\{u_n\}$ follow AP or GP	Show $u_n - u_{n-1} = d$ where d is independent of n.	Show $\frac{u_n}{u_{n-1}} = r$ where r is independent of n.
Special Cases	• When $d = 0$, AP is a constant sequence $\{a, a, a, \ldots\}$. • When $d > 0$, AP is an increasing sequence. • When $d < 0$, AP is a decreasing sequence.	• When $r = 1$, GP is a constant sequence $\{a, a, a, \ldots\}$. • When $r > 0$, the terms are of the same sign. • When $r < 0$, the signs of the terms are alternating.

Word Problem

1. Mr Lee took up a loan of \$10000 on 1st January 2015 from a bank and repays the bank \$$x$ in the middle of every month starting from the month the loan is taken. The bank charges interest at a fixed rate of 5% on the remaining amount owed at the end of each month.

   (i) Show that the amount of money that Mr Lee still owe the bank at the end of n months is given by $\$\left[10000\left(1.05^n\right) - 21\left(1.05^n - 1\right)x\right]$.

   (ii) How much should Mr Lee repay every month so that he can completely repay the bank by the 31st December 2016? Give your answers to the nearest cent.

Answer: (ii) \$690.20

Solutions available on next page…

[Solutions]

(i) At the end of — Amt owed

1st mth: $(\$10000 - x)1.05 = 1.05(10000) - 1.05x$

2nd mth: $[1.05(10000) - 1.05x - x]\,1.05$

$= 1.05^2(10000) - 1.05^2 x - 1.05x$

3rd mth: $[1.05^2(10000) - 1.05^2 x - 1.05x - x]\,1.05$

$= 1.05^3(10000) - 1.05^3 x - 1.05^2 x - 1.05x$

$$= 1.05^3(10000) - x\,(\underbrace{1.05 + 1.05^2 + 1.05^3}_{\text{geometric series with 3 terms and } a = 1.05,\ r = 1.05})$$

n^{th} mth: $1.05^n(10000) - x\,(1.05 + 1.05^2 + 1.05^3 + \cdots + 1.05^n)$

$$= 1.05^n(10000) - \frac{1.05(1.05^n - 1)}{1.05 - 1}\,x$$

$$= 1.05^n(10000) - 21\,(1.05^n - 1)\,x \quad \text{(shown)}$$

(ii) From 1^{st} Jan 2015 to 31^{st} Dec 2016, Mr Lee repays $\$x$ for a total of 24 months, i.e. $n = 24$

Amt owed at end of 24 months $= 1.05^{24}(10000) - 21\,(1.05^{24} - 1)\,x$

To completely repay the bank, amt owed $= 0$.

$$\therefore\ 1.05^{24}(10000) - 21\,(1.05^{24} - 1)\,x = 0$$

$$x = 690.1991$$

$$\approx 690.20$$

Ans : $\$690.20$.